NON-CONED CYCLES: A NEW APPROACH TO TOURNAMENTS

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RESUMO: Neste artigo apresentamos as definições e algumas propriedades básicas dos ciclos não-conados em torneios. Damos a motivação para as definições no contexto da Teoria de Homotopia Regular. Definimos o conceito de característica cíclica de um torneio hamiltoniano, calculando-a para algumas famílias conhecidas de torneios. Apresentamos também algumas aplicações onde o conceito de ciclo não-conado desempenha um papel importante, tais como teoremas de classificação, caracterizações estruturais de certas famílias de torneios e situações nas quais torneios estão associados, por exemplo, a variedades bandeiras complexas de modo a estabelecer se uma estrutura quase complexa invariante nelas admite ou não certas métricas invariantes.

Palavras-chave: Digrafos. Torneios. Homotopia regular. Ciclos conados e não-conados. Característica cíclica. Variedades bandeiras complexas.

ABSTRACT: In this paper we present the definitions and some of the basic properties of the non-coned cycles in tournaments. We give the motivation for the definitions in the context of the Regular Homotopy Theory. We define the concept of cyclic characteristic of a hamiltonian tournament, computing them for some known families of tournaments. We also present some applications where the concept of non-coned cycle plays an important role, such as classification theorems,structural characterizations of certain families of tournaments and situations in which tournaments are associated, for instance, to complex full flag manifolds in order to stablish if an invariant almost complex structure on them admits or not certain type of invariant metrics.

Keywords: Digraphs. Tournaments. Regular homotopy. Coned and non-coned cycles. Cyclic characteristic. Complex flag manifolds.

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1. Introduction

Tournaments are for sure the most well-studied class of digraphs. There are hundreds of research papers written on this subject. In this paper we present the concepts of *coned* and *non-coned cycles*, which seem to provide a new and efficient instrument to study tournaments, specially the hamiltonian ones (which the majority of them). These concepts came up in a natural way in the development of the Regular Homotopy Theory for digraphs. In the case of tournaments, as we intend to show here, the concept of *non-coned cycle* leads to some very interesting and important results.

In section 2 we present the basic definitions, introducing the notation and stating some basic results about digraphs.

In section 3 we recall some of the main concepts and results of the regular homotopy for digraphs as it was introduced by Burzio, Demaria and Garbaccio-Bogin (see [4], [6], and [12]).

In section 4 we define the concepts of *coned* and *non-coned cycle*, giving their motivation in the context of regular homotopy theory.

In section 5 we show how the concept of coned cycle can be used in several situations if one is interested in classification results or in structural characterizations of certain families of tournaments and also, as it has been seen lately, in those situations in which tournaments are associated to complex full flag manifolds in order to stablish if an invariant almost complex structure on them admits or not certain type of invariant metrics. .

2. Preliminaries

In this section we give some definitions and recall some well known results on digraphs.

Definition 2.1 Let V be a finite non-empty set and E a set of ordered pairs $(u, v) \in V \times V$, such that $u \neq v$. We call the pair $D = (V, E)$ a *directed graph* or *digraph*. The elements of V are the *vertices* of D, the cardinality of V the *order* of D, and the elements of E the *arcs* of D. Moreover we write $u \to v$ instead of (u, v), and we call u a *predecessor* of v and v a *successor* of u.

Remark 1. Given two distinct vertices u and v we have *a priori* four possibilities, and then four types of arcs:

(1) There is no oriented arc between u and v, and then we shall denote $u|v$ the *null arc*;

(2) there is the oriented arc (u, v), but not the arc (v, u), and then we shall denote the *simple arc* $u \to v$;

(3) there is the oriented arc (v, u), but not the arc (u, v), and then we shall denote the *simple arc* $u \leftarrow v$;

(4) there are both oriented arcs (u, v) and (v, u), and then we shall denote the *double arc* $u \leftrightarrow v$. (A double arc is also called a *2-cycle* or a *symmetric pair*.)

Definition 2.2 A digraph is called oriented if, between two distinct vertices, there is at most one ordered arc – that is, the possible arcs are either simple arcs or null arcs; a digraph is called a *non-oriented graph* if, between two distinct vertices, there is either a double arc or a null arc. A digraph is called *semicomplete* if, between two distinct vertices, there is at least one ordered arc; the possible arcs in this case are either simple or double arcs.

Definition 2.3 A digraph T is a *tournament* if, between every pair of distinct vertices, there is one and only one arc. A tournament T is called *hamiltonian* if it contains a spanning cycle – that is a cycle passing through all vertices of T. If the cardinality $|V(T)| = n$, we say that T has order n. For the sake of brevity we will identify T_n with its vertex set $V(T_n) = \{v_1, \ldots, v_n\}$. We denote by $T_n - v$ the *vertex deleted tournament* and by $< C >$ the subtournament $< V(C) >$ induced by the vertices of the cycle C of T. Moreover $A \to B$ denotes that the vertices of a subtournament A are predecessors of the vertices of the subtournament B in T_n.

Definition 2.4 A vertex v *cones* a subtournament T' in T_n (or equivalently, T' is *coned* by v) if and only if either $v \to T'$ or $T' \to v$. Otherwise T' is said to be *non-coned* in T_n. If C is a cycle in T_n, we say that C is *coned* by v (or equivalently, v *cones* C) if $< C >$ is coned by v.

Definition 2.5 A subtournament S of T is an *e-component* of T, and its vertices are all called *equivalent*, if S is coned by each vertex of $T \setminus S$. Single vertices of T are trivial e-components.

We have the notion of *quotient tournaments* (see [24]), that can also be described in the following way: Every tournament T_n can be partitioned (in a non-trivial way) into disjoint e-components $S^{(1)}, S^{(2)}, \ldots, S^{(m)}$. These components can be considered as the vertices $w_1, w_2, \ldots, w_m$ of a tournament R_m, so that T_n can be obtained as the *composition* $R_m(S^{(1)}, S^{(2)}, \ldots, S^{(m)})$ of the e-components $S^{(1)}, S^{(2)}, \ldots, S^{(m)}$ with the *quotient* R_m. In other words $T_n = S^{(1)} \cup S^{(2)} \cup \cdots \cup S^{(m)}$ and $a \rightarrow b$ in T_n if and only if $a \rightarrow b$ in some $S^{(j)}$ or $a \in S^{(j)}$, $b \in S^{(k)}$ and $w_j \rightarrow w_k$ in R_m (i.e. $S^{(j)} \rightarrow S^{(k)}$).

A tournament T_n is *simple* if it has no non-trivial e-components. In other words, if $T_n = R_m(S^{(1)}, S^{(2)}, \ldots, S^{(m)})$ is simple then either m = n or m = 1.

A subtournament T' of T is called *shrinkable* if it is included in a non-trivial e-component of T_n. If $< C >$ is shrinkable in T_n, then we say the cycle C is *shrinkable* in T_n (see [5]).

We recall the following results about quotient tournaments:

Proposition 2.6 The quotient tournament R_m is isomorphic to a subtournament of T_n.

Proposition 2.7 Every tournament T_n, with $n \geq 2$, admits exactly one simple quotient tournament.

Proposition 2.8 A tournament T_n is hamiltonian if, and only if, every one of its quotient tournaments is hamiltonian.

3. Regular Homotopy for Digraphs

In this section we recall some of the basic concepts and results on the regular homotopy for digraphs (see [4], [6] and [12]).

We can, in a natural way, consider a digraph D as a pre-topological space (Čech closure space) $P(D)$. We just have to consider the principal filter generated by the closed neighbourhoods $N[v] = \{v\} \cup \{w \in D | v \rightarrow w\}$ as the neighbourhoods filter of any vertex v of D. Then introducing precontinuous functions and a homotopy theory for digraphs, similar to the classical one for topological spaces, can be developed (see [12]). The *precontinuous* maps from a topological space into the pretopological space $P(D)$ are the *O-regular* maps considered in [2], [3] or [4].

In [4] Burzio and Demaria proved that the regular homotopy groups Q_n of $P(D)$ are isomorphic to the classical homotopy groups π_n of the polyhedron of a suitable simplicial complex K_D associated with D. That is

$$Q_n(D, v) \cong \pi_n(|K_D|, v) \quad .$$

We shall describe shortly how to get this suitable simplicial complex K_D.

If $H \subset D$ we say H is *headed* if there exists a vertex v in H such that $v \to H \setminus v$. And H is said to be *totally headed* if for every $A \subset H$ with $A \neq \emptyset$, we have that A is headed. The simplexes in K_D are the ones generated by the totally headed subdigraphs of D.

4. Coned cycles in a Tournament

In this section we shall take a closer look at the concept of coned cycles in a tournament T. We will show how this concept relates to some of the homotopical properties of T.

First of all, we recall (see [6]) that for a tournament T the *simplicial complex* K_T associated is such that its 0-simplices are given by the vertices ot T, and all other simplices are given as follows: $S = (v_1, \ldots, v_n) \in K_T$ if and only if the subtournament $< v_1, \ldots, v_n >$ induced by the vertices $v_1, \ldots, v_n$ is transitive.

As it is stated in the previous section, the regular homotopy n-dimensional group $Q_n(T)$ of the tournament T is isomorphic to the classical homotopy group $\Pi_n(|K_T|)$ of the polyhedron $|K_T|$ associated to the simplicial complex K_T.

Let a cycle $C : v_1 \to v_2 \to \cdots \to v_r \to v_1$ be coned by a vertex v in T; let, for instance $C \to v$.

We can see that all the subtournaments $S^i =< v_i, v_{i+1}, v >$ with v_i, v_{i+1} in C, are transitive. Therefore $(v_i, v_{i+1}, v) \in K_T$ and hence the cycle C is nullhomotopic.

In the case C is contained in an e-component of T, then C is coned, hence nullhomotopic, and we say the cycle C is *shrinkable*. This fact that all the coned cycles are nullhomotopic in the context of the regular homotopy for digraphs has called attention to the non-coned cycles, whose concept was essential to prove very significant new results about hamiltonian tournaments as we shall show in the several applications that are given in the next section.

5. Some Applications

In [6] Burzio and Demaria used regular homotopic methods for characterizing tournaments which are the composition of a non-trivial highly regular tournament.

Definition 5.1 A tournament T_{2m+1} is *highly regular* if we can label its vertices $v_1, v_2, \ldots, v_{2m+1}$ in such a way that $v_i \to v_j$, for all $i = 1, 2, \ldots, 2m+1$ and for all $j = i+1, i+2, \ldots, i+m(\bmod 2m+1)$. In other words, T_{2m+1} is highly regular if there is a cyclic ordering $v_1, v_2, \ldots, v_{2m+1}, v_1$ of its vertices such that $v_i \to v_j$ if and only if v_j is one of the first m successors of v_i in the cyclic ordering of the vertices of T_{2m+1}.

Definition 5.2 A tournament T is said to be *simply disconneceted* if its fundamental group $Q_1(T)$ is not trivial.

Burzio and Demaria characterized the simply disconnected tournaments proving the following result:

Theorem 5.1 A tournament T is simply disconnected if and only if T is the composition of any tournaments with a non-trivial highly regular tournament (i.e. $T_n = R_k(S^{(1)}, \ldots, S^{(k)})$, where $S^{(i)}$ are arbitrary tournaments and R_k is a highly regular tournament).

(See Theorem 3.9, in [6]).

They also gave a different characterization for the tournaments which are simply disconnected, proving the following:

Theorem 5.2 A tournament T is simply disconnected if and ony if

(a) there exists a non-coned 3-cycle;
(b) all the coned 3-cycles are shrinkable.

(See Theorem 7, in [5]).

Remark 5.1 It was in this paper that Burzio and Demaria introduced for the first time the concept of coned and non-coned 3-cycles, which was later on generalized to cycles and sub-tournaments.

Definition 5.3 Let H_n be a hamiltonian tournament. A vertex v of H_n is called a *neutral vertex* of H_n if $H_n \setminus v$ is hamiltonian. The number of the neutral vertices of H_n is denote by $\nu(H_n)$.

Remark 5.2 We observe that $\nu(H_n)$ is also the number of hamiltonian subtournaments of order $n-1$, so we have that $\nu(H_n) \leq n$, for we can have at most n subtournaments of order $n-1$. On the other hand, in [26] Moon proved that the minimum number of k-cycles, with $3 \leq k \leq n$, in a hamiltonian tournament H_n is equal to $n-k+1$. Hence we have that $\nu(H_n) \geq 2$, if $n \geq 4$. Therefore, we have that $2 \leq \nu(H_n) \leq n$, for $n \geq 4$.

Another application of the concept of non-coned cycle was given in [7]. We have the following:

Theorem 5.3 A tournament H_n $(n \geq 5)$ is hamiltonian if and only if there exists an m-cycle C, with $3 \leq m \leq n-2$, which is non-coned in H_n.

Proof:

Let us suppose H_n is hamiltonian. Let v be a neutral vertex of H_n and v_1, v_2 two neutral vertices of $H_n \setminus v$. Let us suppose by contradiction that the two hamiltonian subtournaments $H_n \setminus \{v, v_1\}$ and $H_n \setminus \{v, v_2\}$ are both coned. Since v_1 cannot cone $H_n \setminus \{v, v_1\}$ (otherwise $H_n \setminus \{v, v_1\}$ is not hamiltonian), and v_2 cannot cone $H_n \setminus \{v, v_2\}$ (otherwise $H_n \setminus \{v, v_2\}$ is not hamiltonian), then both $H_n \setminus \{v, v_1\}$ and $H_n \setminus \{v, v_2\}$ are coned by v. Hence v cones $H_n \setminus v$, which is a contradiction since H_n is hamiltonian. Therefore at least one of the two subtournaments $H_n \setminus \{v, v_1\}$ and $H_n \setminus \{v, v_2\}$ is non-coned. In other words, in H_n there exists at least one non-coned $(n-2)$-cycle. Conversely, if T_n is not hamiltonian, then its simple quotient is T_2. Hence every cycle of T_n is included in an e-component, and therefore it is coned.

Remark 5.3 Also H_3 and H_4 contain non-coned m-cycles, but in this case the condition $m \leq n-2$ is not satisfied.

If C is a non-coned cycle of H_n and $v \notin V(C)$, then it is possible to extend C to a cycle through all the vertices of $H_n \setminus v$. This fact motivated Burzio and Demaria (see [7]) to define:

Definition 5.4 Let C be a non-coned cycle of H_n. The set $P_c = V(H_n) \setminus V(C)$ consists of neutral vertices of H_n, and these are called *poles* of C. A non-coned cycle C of H_n is said to be *minimal* if every cycle C', such that $V(C') \subset V(C)$, is coned by at least one vertex of H_n. A minimal cycle is said to be *characteristic* if it possesses the shortest length of the minimal cycles. The length of a characteristic cycle is called the *cyclic characteristic* of H_n and it is denoted by $cc(H_n)$. The difference $n - cc(H_n)$ is called the *cyclic difference* of H_n and is denoted by $cd(H_n)$.

We observe that if C is a characteristic cycle of H_n, then $cd(H_n) = |P_c|$.

Using these definitions and the result given in Theorem 5.3, Burzio and Demaria in [7] gave a classification for the collection $\mathcal{H}_n$ of all the hamiltonian tournaments of order $n \geq 5$, subdividing it in $n-4$ different classes. Namely, the first class of cyclic characteristic 3 is formed by the tournaments which contain a non-coned 3-cycle; the second one of cyclic characteristic 4 consists of the tournaments which contain a non-coned 4-cycle and whose 3-cycles are all coned. And so on, till the (n-4)th class of cyclic characteristic (n-2) which consists of the tournaments containing a non-coned $(n-2)$-cycle and whose cycles with lower length are all coned.

Formally we have the following:

Theorem 5.4 Let H_n, with $n \geq 5$, be a hamiltonian tournament, then $2 \leq cd(H_n) \leq n-3$ (or equivalently $3 \leq cc(H_n) \leq n-2$). Conversely, for every $n \geq 5$ and for every h such that $2 \leq h \leq n-3$, there exist hamiltonian tournaments H_n with $cd(H_n) = h$.

This classification theorem for the hamiltonian tournaments has led to some important recent results. This is due to the fact this new invariant $cc(H_n)$ (cyclic characteristic) can be obtained in a combinatorial way (just using the adjacency data) other then having some nice properties, like the one given in the next proposition.

Proposition 5.1 If a tournament T_n is the composition $R_m(S^{(1)}, S^{(2)}, \ldots, S^{(m)})$, then $cc(T_n) = cc(R_m)$.

These last two results have allowed to obtain the structural characterization of some important classes of hamiltonian tournaments as one can see by results we present now.

In 1989 Demaria and Gianella defined T_n to be a *normal tournament* if it is hamiltonian and has a unique characteristic cycle. In [11] they throughly studied this class of tournaments, which turned out to be very important in some structural characterization theorems for other classes of tournaments.

We present here some of the most important properties of the normal tournaments.

Definition 5.5 The tournament A_n, with $n \geq 4$, such that $V(A_n) = \{a_1, a_2, \ldots, a_n\}$ and $V(A_n) = \{a_i \to a_j \mid j < i-1 \text{ or } j = i-1\}$ is called the bineutral tournament of order n.

Remark 5.4 It is known that A_n is the unique tournament having exactly two neutral vertices. Moreover, for $n \geq 5$, $\{a_{n-1}, a_n, a_1, a_2\}$ is its maximal transitive subtournament, formed by consecutive vertices of the hamiltonian cycle. It is easy to see that $a_2, \to \cdots \to a_{n-1} \to a_2$ is the only minimal cycle (the *characteristic* one).

We also observe that $cc(A_4) = 3$, with two minimal cycles, so that it is not normal.

In [13] Demaria and Gianella have also shown that a normal tournament H_n has as its characteristic cycle either the 3-cycle or a bineutral tournament A_k $(n \geq 4)$. In the same paper they have proofed the following:

Proposition 5.2 Let H_n be a normal tournament with cyclic characteristic k ($k \geq 3$) and let A_k be its characteristic cycle. A pole z, associated to A_k, must have the following adjacencies with respect to A_k:

1) $(a_{i+1}, a_{i+2}, \ldots, a_k) \to z \to (a_1, a_2, \ldots, a_i)$ $(1 \leq i \leq k-1)$.
2) $(a_i, a_{i+2}, a_{i+3}, \ldots, a_k) \to z \to (a_1, \ldots, a_{i-1}, a_{i+1})$ $(1 \leq i \leq k-1)$.

Definition 5.6 The pole z is called a *pole of kind i and class* 1 *or class* 2 (and denoted by x_i or y_i) if its adjacencies are given by the previous conditions 1) or 2), respectively.

This class of the normal tournaments is very important in the study of the hamiltonian tournaments, for instance, the class of the hamiltonian tournaments which have a unique n-cycle, which was characterized by Douglas (see [18]), can now be characterized in a different way as it is shown in the next proposition.

Proposition 5.3 Let H_n be a hamiltonian tournament with $cc(H_n) = k \geq 3$. H_n is a Douglas tournament if, and only if:

1.1) H_n has as a simple quotient Q_m $(m \geq 5)$ such that:

a) Q_m is normal;

b) the subtournament of the poles in Q_m is transitive;

c) the poles of Q_m are all of class 1;

d) between two poles x_i and $x_j^{'}$ of Q_m of class 1, the following rules of adjacies hold $x_i \to x_j^{'}$ implies $j \leq i+1$.

1.2) H_n can be constructed from Q_m by replacing all the vertices of Q_m, but the vertices $a_2, \ldots, a_{k-1}$ of its characteristic cycle A_k, by some transitive tournament.

2) H_n is the composition of a singleton and two transitive tournaments with a 3-cycle.

Proof:

(See [14])

Later Demaria and Kiihl, using this characterization and the structural characterization of the normal tournaments given in [13] (by Demaria and Gianella), obtained the enumeration of the Douglas tournaments with a convenient variation of the Pascal triangle (see [17]).

Later Demaria, Guido and others (see [14], [19], [20] and [21]) have used the concept of non-coned cycles in order to approach the reconstruction problem for tournaments. It is known (see [30] and [31]) the reconstruction conjecture fails for tournaments. Then the challenge is to find a characterization (if any) of reconstructable or non-reconstructable tournaments. In this context the reconstruction of combinatorial properties and invariants of tournaments certainly are very useful. In [21] Guido and Kiihl computed the cyclic characteristic of all known tournaments which are non-reconstructable. It is early to say but it seems there might be some direct link between reconstructable hamiltonian tournaments and their cyclic characteristic. In fact, as poit out in [21], no non-reconstructable tournament is known having $cc(H) > 4$.

Recently we have seen situations in which tournaments are associated to complex full flag manifolds in order to stablish if an invariant almost complex structure on them admits or not certain type of invariant metrics (see [23]). Mo and Negreiros (see [24]) have shown that a necessary condition for an invariant almost complex structure on the complex full manifold $F(n)$ to admit a (1,2)-symplectic invariant metric is that its associated tournament be *cone-free*.

A tournament T_n is called *cone-free*, by Mo and Negreiros, if its restriction to any four vertices subtournament is never a coned 3-cycle, that is, all 3-cycles on T_n are non coned.

Later (see [10] Cohen, Negreiros and San Martin have shown that the cone-free condition on the associated tournament is also sufficient.

This approach to study invariant metrics on flag manifolds, using its associated tournament and the concept of non-coned 3-cycles, is very effective and it has yield important results (see [11] and [29]).

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